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Reply

Reply to A. Zhong "Discussions on 'A constitutive model for the Mullins effect with permanent set in a particle-reinforced rubber' by A. Dorfmann and R.W. Ogden"

We thank Dr. Zhong for his comments on the paper 'A constitutive model for the Mullins effect with permanent set in a particle-reinforced rubber' by Dorfmann and Ogden (2004). In our email correspondence with Dr. Zhong we have attempted to explain his errors and misinterpretations, but, unfortunately, he has nevertheless chosen to perpetuate these errors in print.

On one issue we are agreed. Stress softening and permanent set effects in rubberlike solids are extremely complex and their modeling is very difficult. Our paper is an attempt to model *limited aspects* of these effects. It is therefore invidious for Dr. Zhong to criticize the paper for not being able to model what we were not attempting to model. Thus, most of his comments are irrelevant. Nevertheless, a few remarks are in order.

First, he refers to our equations

$$\frac{\partial W}{\partial \eta_i}(\mathbf{F}, \eta_1, \eta_2) = 0, \quad i = 1, 2, \tag{1}$$

as being ad hoc assumptions and as having no physical meaning. He is incorrect on both counts. In the static context, for instance, the equations can be formulated in variational form, in which case vanishing of the first variation of the total energy leads immediately to (1), which can then be thought of as optimization conditions giving the 'best' choice of η_i as functions of $\mathbf{F}$. For the case of a single variable η Eq. (1) has been derived in the paper by Ogden (2001), cited below, in which the optimization condition interpretation was discussed, and we refer Dr. Zhong to this article for the details, which carry over with minor modifications to the case of (1). In a more general setting in which the η_i are not constrained to depend on $\mathbf{F}$ and are independent 'internal' variables requiring an evolution equation for their description, the expressions on the left-hand side in (1) would be non-zero and then represent 'internal forces'.

Dr. Zhong refers to the illustrative example considered as a 'questionable example' on the basis that it has 'too many' material parameters. However, the considered complex behavior requires a model that can

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capture, firstly and independently, the virgin curve, secondly the unloading characteristics of stress softening, thirdly the permanent set, and fourthly the anisotropy. In combination these behaviors are very complicated and necessarily require a significant number of parameters, whatever type of model is used. Dr. Zhong's criticism would therefore apply to any model that attempts to simulate the behavior of the considered materials. Dr. Zhong complains that 'there are too many adjustable parameters and functions'—in fact, there is only one function involved in the model, v_1, v_2, v_3 being constants, not functions—and 'which can be determined arbitrarily to some degree'—a gratuitous comment without any rational foundation.

Dr. Zhong suggests that the material constants for the Ogden model ($M = 3$) in Section 6.1 are unstable. This is a meaningless statement since material constants cannot be unstable, although the material response (based on use of those constants) could be unstable. In fact, it is not clear from his text what Dr. Zhong means by instability, whether it is material instability or numerical instability, for example. In either case he is incorrect. In his email correspondence Dr. Zhong mentioned that he had used ABAQUS for his calculations, but he appears to have misused it—and his conclusions are incorrect. In fact, the material parameters satisfy mathematical conditions that guarantee material stability. The curves in our Figs. 9 and 10 were obtained for the parameters in Table 1 using Matlab and they have been checked very carefully from various standpoints. Beware of black boxes!

References

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